NAG Toolbox for MATLAB

f07ad

1 Purpose

f07ad computes the LU factorization of a real m by n matrix.

2 Syntax

```
[a, ipiv, info] = f07ad(a, 'm', m, 'n', n)
```

3 Description

f07ad forms the LU factorization of a real m by n matrix A as $A = PLU$, where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and U is upper triangular (upper trapezoidal if $m < n$). Usually A is square ($m = n$), and both L and U are triangular. The function uses partial pivoting, with row interchanges.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda,**$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n coefficient matrix A.

5.2 Optional Input Parameters

1: **m – int32 scalar**

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda

5.4 Output Parameters

1: **a**(**lda,**$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The factors L and U from the factorization $A = PLU$; the unit diagonal elements of L are not stored.

2: **ipiv**$(*)$ – **int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

The pivot indices. Row i of the matrix A was interchanged with row **ipiv**(i), for $i = 1, 2, \ldots, \min(m, n)$.

3: **info** – **int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **a**, 4: **lda**, 5: **ipiv**, 6: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, $U(i, i)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(\min(m, n))\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$ if $m = n$ (the usual case), $\frac{1}{3}n^2(3m - n)$ if $m > n$ and $\frac{1}{3}m^2(3n - m)$ if $m < n$.

A call to this function with $m = n$ may be followed by calls to the functions:

f07ae to solve $AX = B$ or $A^{\mathrm{T}}X = B$;

f07ag to estimate the condition number of A;

f07aj to compute the inverse of A.

The complex analogue of this function is f07ar.

9 Example

```
a = [1.8, 2.88, 2.05, -0.89;
     5.25, -2.95, -0.95, -3.8;
     1.58, -2.69, -2.9, -1.04;
     -1.11, -0.66, -0.59, 0.8];
```

```
[aOut, ipiv, info] = f07ad(a)
```

```
aOut =
    5.2500   -2.9500   -0.9500   -3.8000
    0.3429    3.8914    2.3757    0.4129
    0.3010   -0.4631   -1.5139    0.2948
   -0.2114   -0.3299    0.0047    0.1314
ipiv =
           2
           2
           3
           4
info =
           0
```